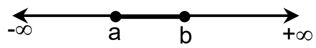
LINEAR INEQUATIONS

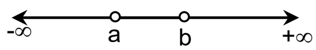
Intervals

(a) Closed Interval. Let a and b be two real numbers, where a < b. Then the set of all real numbers x such that $a \le x \le b$ is called closed interval, which is denoted by [a, b].
Thus $[a, b] = \{x \in R: a \le x \le b\}$.
Graphically: On the real line, [a, b] is as shown below:



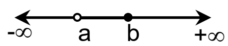
Here end points a and b are included, which are shown by closed circles.

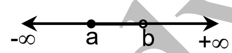
(b) Open Interval. Let a and b be two real numbers, where a < b. Then the set of all real numbers x such that $a < x < b$ is called open interval, which is denoted by (a, b).
Thus $(a, b) = \{x \in R: a < x < b\}$.
Graphically: On the real line, (a, b) is as shown below:



Here end points a and b are excluded, which are shown by open circles.

(c) Semi-closed/Semi-open Intervals. Let a and b be two real numbers, where a < b. Then the set of all real numbers x such that $a < x \le b$ or $a \le x < b$ are called semi-closed/semi open intervals.
Thus $(a,b] = \{x \in R: a < x \le b\}$ and $[a, b) = \{x \in R: a \le x < b\}$.
Graphically : On the real line, (a, b] and [a, b) are as shown below:





Inequations

Definition: A statement, which involves variable (s) and the sign of inequality viz. >, <, ≥ or ≤ is called an inequation.
For example:

(i) $ax + b < 0$, $ax + b \le 0$, $ax + b > 0$, $ax + b \ge 0$ ($a \ne 0$) are inequation in one variable x.

(ii) $ax + by < c$, $ax + by \le c$, $ax + by > c$, $ax + by \ge c$ ($a \ne 0$, $b \ne 0$) are inequations in two variables x and y.

(iii) $ax^2 + bx + c < 0$, $ax^2 + bx + c \le 0$, $ax^2 + bx + c > 0$, $ax^2 + bx + c \ge 0$ ($a \ne 0$)are quadratic inequations in one variable x.

Solutions of Inequations

Definition: A solution of an inequation is the value (s) of the variable (s), which makes it a true statement.

General Rules

There are some general rules for solving an inequation algebraically, which are similar to those for solving an equation. These are rules are given below :

Rule I. We can add (or subtract) the same number to (or from) both sides of an inequation.
Thus $a \le b \Rightarrow a \pm x \le b \pm x$.

Rule II. We can multiply (or divide) both sides of an inequation by a positive number.
Thus $a \le b \Rightarrow 3a \le 3b$.
If we multiply both sides of an inequation by a negative number, the signs of inequality viz. '<' and '>' are reversed.
Thus $a \le b \Rightarrow -2a \ge -2b$ and $a \ge b \Rightarrow -2a \le -2b$.

Rule III. If $\frac{a}{b} \geq \frac{c}{d}$, then

(i) ad ≥ bc if b and d are of same signs and

(ii) ad ≤ bc if b and d are of opposite signs.

Thus we can multiply both sides of $\frac{a}{b} \geq \frac{c}{d}$ by bd without changing the sign of inequation when bd is positive i.e. when b and d are of same signs. The sign of inequation is reversed when bd is negative i.e. when b and d are of opposite signs.

Rule IV. We cannot take the square roots of an inequation in the same way we take square roots of an equation.

Thus $x^2 = 9 \Rightarrow x = \pm 3$ but $x^2 \leq 9 \not\Rightarrow x \leq \pm 3$

Rather we have:

$x^2 \leq a^2 \Rightarrow |x| \leq a \Rightarrow -a \leq x \leq a$ and $x^2 \geq a^2 \Rightarrow |x| \geq a \Rightarrow x \leq -a$ or $x \geq a$.

Linear Inequations

Definition: An inequation is said to be linear if each term of the algebraic expression (or expressions) of the inequation contains variables of first degree and does not contain the term involving the product of the variables.

Thus $ax + b < 0$, $ax + b \leq 0$, $ax + b > 0$, $ax + b \geq 0$ $(a \neq 0)$ are linear inequations in one variable x and $ax + by + c < 0$, $ax + by + c \leq 0$, $ax + by + c > 0$, $ax + by + c \geq 0$ $(a \neq 0, b \neq 0)$ are linear inequations in two variables x and y.

Solutions of Linear Inequations in one variable

We adopt the following algorithmic approach to solve the linear inequations in one variable:

Algorithmic Approach

(i) Write the given linear inequation.

(ii) Collect all terms involving the variable on LHS and constant terms on RHS.

(iii) Simplify both sides so as to get inequation of the form $ax < b$ or $ax \leq b$ or $ax > b$ or $ax \geq b$.

(iv) Solve the inequations obtained in step (iii) by dividing both sides by the coefficient of the variable.

(v) Write the solution set in the form of an interval or represent it on the real line.

Illustration 1

Solve the inequation 4x –7 < 3 –x

(i) when x is an integer (ii) when x is a natural number.

Given, $4x - 7 < 3 - x$

$\Rightarrow 4x - 7 + 7 < 3 - x + 7$ [Adding 7 to both sides]

$\Rightarrow 4x < 10 - x$

$\Rightarrow 4x + x < 10 - x + x$

[Adding x to both sides or transferring –x from right side to left side]

$\Rightarrow 5x < 10$

$\Rightarrow x < 2$ [Dividing by both sides by 5]

(i) When x is an integer, x = 1, 0, – 1, –2, –3, …..

(ii) When x is a natural number, x = 1

Illustration 2

If $x \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, solve the inequation $-2x + 6 \leq 5x - 4$.

Given, $-2x + 6 \leq 5x - 4$

$\Rightarrow \quad -2x + 6 - 6 \leq 5x - 4 - 6$ [Subtracting 6 from both sides]

$\Rightarrow \quad -2x \leq 5x - 10$

$\Rightarrow \quad -2x - 5x \leq 5x - 10 - 5x$ [Subtracting 5x from both sides]

$\Rightarrow \quad -7x \leq -10$

$\Rightarrow \quad x \geq \frac{10}{7}$

[Dividing both sides by –7, here inequality has changed because we have divided by a negative number]

$\therefore \quad x \in \{0, 1, 2, \ldots\ldots, 10\}$ $\therefore$ $x = 2, 3, 4, 5, 6, 7, 8, 9, 10$.

Illustration 3

Solve the following inequation:

$5x - 15 \geq 0$.

The given inequation is $5x - 15 \geq 0$ …(1)

$\Rightarrow \quad 5x \geq 15$ [Transposing 15 to RHS]

$\Rightarrow \quad \frac{5x}{5} \geq \frac{15}{5}$ [Dividing both sides by 5]

$\Rightarrow \quad x \geq 3$.

Hence the solution set is $[3, \infty)$, which is shown on the real line as below:

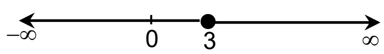


Illustration 4

Solve the following inequation:

$5x - 1 > 3x + 7$.

The given inequation is $5x - 1 > 3x + 7$

$\Rightarrow \quad (5x - 1) + 1 > (3x + 7) + 1$ [Adding 1 on both sides]

$\Rightarrow \quad 5x > 3x + 8$

$\Rightarrow \quad 5x - 3x > (3x + 8) - 3x$ [Adding – 3x on both sides]

$\Rightarrow \quad 2x > 8$

$\Rightarrow \quad \frac{2x}{2} > \frac{8}{2}$ [Dividing both sides by 2]

$\Rightarrow \quad x > 4$.

Hence the solution set is $(4, \infty)$, which is shown on the real line as below:

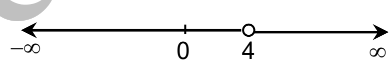


Illustration 5

Solve the inequation $37 - (3x + 5) \geq 9x - 8(x - 3)$

Given, $37 - (3x + 5) \geq 9x - 8(x - 3)$

$\Rightarrow \quad 37 - 3x - 5 \geq 9x - 8x + 24$

$\Rightarrow \quad 32 - 3x \geq x + 24$

$\Rightarrow$ $32 - 3x - 32 \geq x + 24 - 32$ [Subtracting 32 from both sides]

$\Rightarrow$ $-3x \geq x - 8$

$\Rightarrow$ $-3x - x \geq - 8$ [Transferring x from right side to left side]

$\Rightarrow$ $-4x \geq - 8$

$\Rightarrow$ $x \leq 2$ [Dividing both sides by –4]

$\therefore$ Solution set of given inequation = $(-\infty, 2]$.

Solution set on number line is as given in the figure

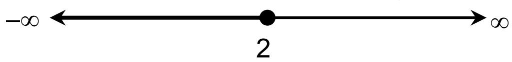


Illustration 6

Solve the following inequation:

$$\frac{3(x-2)}{5} \geq \frac{5(2-x)}{3}.$$

The given inequation is $\frac{3(x-2)}{5} \geq \frac{5(2-x)}{3}$

$\Rightarrow$ $\frac{3x-6}{5} \geq \frac{10-5x}{3}$

$\Rightarrow$ $3(3x - 6) \geq 5(10-5x)$ [Multiplying both sides by 15 i.e. the l.c.m. of 5 and 3]

$\Rightarrow$ $9x - 18 \geq 50 - 25x$

$\Rightarrow$ $9x + 25x \geq 50 + 18$ [Transposing – 25x to LHS and – 18 to RHS]

$\Rightarrow$ $34x \geq 68$

$\Rightarrow$ $\frac{34x}{34} \geq \frac{68}{34}$ [Dividing both sides by 34]

$\Rightarrow$ $x \geq 2$.

Hence the solution set is $[2, \infty)$, which is shown on the real line as below:

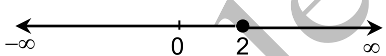


Illustration 7

Solve the following inequation:

$$\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}.$$

The given inequation is $\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$.

$\Rightarrow$ $\frac{x}{4} < \frac{5(5x-2)-3(7x-3)}{15}$

$\Rightarrow$ $\frac{x}{4} < \frac{25x-10-21x+9}{15}$ $\Rightarrow$ $\frac{x}{4} < \frac{4x-1}{15}$

$\Rightarrow$ $15x < 4\,(4x - 1)$ [Multiplying both sides by 60 i.e. l.c.m. of 4 and 15]

$\Rightarrow$ $15x < 16x - 4$

$\Rightarrow$ $15x - 16x < - 4$ [Transposing 16x to LHS]

$\Rightarrow$ $-x < -4$

$\Rightarrow$ $x > 4$. [Multiplying both sides by (–1)]

Hence the solution set is $(4, \infty)$, which is shown on the real line as below:

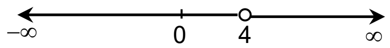


Illustration 8

Solve the following inequation:

$$\frac{x+3}{x-2} \le 2$$

$\frac{x+3}{x-2} - 2 \le 0 \qquad \Rightarrow \qquad \frac{(x+3)-2(x-2)}{x-2} \le 0$

$\Rightarrow \qquad \frac{x+3-2x+4}{x-2} \le 0 \qquad \Rightarrow \qquad \frac{7-x}{x-2} \le 0$

$\Rightarrow \qquad \frac{x-7}{x-2} \ge 0$ [Multiplying both sides by (–1)]

$\therefore$ either $x - 7 \ge 0,\ x - 2 > 0$ or $x - 7 \le 0,\ x - 2 < 0$

i.e. either $x \ge 7,\ x > 2$ or $x \le 7,\ x < 2$

i.e. either $x \ge 7$ or $x < 2$ [$\because x \ge 7 \Rightarrow x > 2$ and $x < 2 \Rightarrow x \le 7$]

i.e. either $x \in [7, \infty)$ or $x \in (-\infty, 2)$.

Hence the solution set is $(-\infty, 2) \cup [7, \infty)$ which is shown on the real line as below:

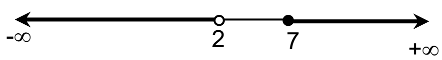


Illustration 9

Solve the inequation $\frac{x+8}{x+2} > 1$

Given, $\frac{x+8}{x+2} > 1$

$\Leftrightarrow \qquad \frac{x+8}{x+2} - 1 > 0$

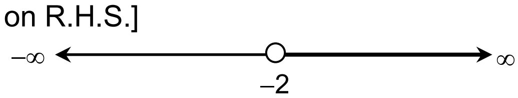
$\Leftrightarrow \qquad \frac{x+8-x-2}{x+2} > 0$

$\Leftrightarrow \qquad \frac{6}{x+2} > 0$

$\Leftrightarrow \qquad x + 2 > 0$ $\left[\because \frac{a}{b} > 0 \text{ and } a > 0 \Rightarrow b > 0\right]$

$\Leftrightarrow \qquad x > -2$ [Transferring 2 on R.H.S.]

$\therefore$ Solution set = $(-2, \infty)$.



Solution set on the number line is as given in the figure (shown by thick line).

Illustration 10

Solve: $\frac{2x-3}{4} + 8 \ge 2 + \frac{4x}{3}$.

The given inequation is $\frac{2x-3}{4} + 8 \ge 2 + \frac{4x}{3}$

$\Rightarrow \quad 12\left(\frac{2x-3}{4}+8\right) \geq 12\left(2+\frac{4x}{3}\right)$

[Multiplying both sides by 12 i.e. L.C.M of 4 and 3]

$\Rightarrow \quad 3(2x-3) + 96 \geq 24 + 16x$

$\Rightarrow \quad 6x - 9 + 96 \geq 24 + 16x$

$\Rightarrow \quad 6x + 87 \geq 24 + 16x$

$\Rightarrow \quad 6x - 16x \geq 24 - 87$ [Transposing 16x to LHS and 87 to RHS]

$\Rightarrow \quad -10x \geq -63$

$\Rightarrow \quad \frac{-10x}{-10} \leq \frac{-63}{-10}$ [Dividing both sides by – 10]

$\Rightarrow \quad x \leq 6.3.$

Hence the solution set is $(-\infty, 6.3]$, which is shown on the real line as below:

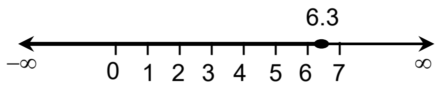


Illustration 11

Solve 3x + 5 < x – 7, when

(i) x is an integer (ii) x is a real number.

The given inequation is $3x + 5 < x - 7$ …(1)

$\Rightarrow \quad (3x+5) - 5 < (x-7) - 5$ [Adding –5 on both sides]

$\Rightarrow \quad 3x < x - 7 - 5$

$\Rightarrow \quad 3x < x - 12$

$\Rightarrow \quad 3x - x < -12$ [Transposing x to LHS]

$\Rightarrow \quad 2x < -12$

$\Rightarrow \quad \frac{2x}{2} < \frac{-12}{2}$

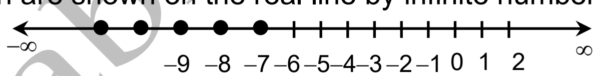
$\Rightarrow \quad x < -6.$ [Dividing both sides by 2]

(i) When x is an integer.

Here the solutions of (1) are ……..–9, –8, –7.

∴ The solution set is {……–9, –8, –7},

Which are shown on the real line by infinite number of points as below:



(ii) When x is a real number.

Here solution set is $(-\infty, -6)$, which is shown on the real line as below:

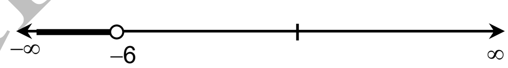
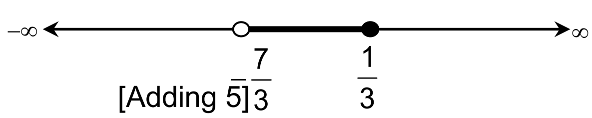


Illustration 12

Solve the inequation –12 < 3x – 5 ≤ –4.

Given , $-12 < 3x - 5 \leq -4$

$\Rightarrow \quad -12 + 5 < 3x - 5 + 5 \leq -4 + 5$ [Adding 5]



$\Rightarrow \quad -7 < 3x \le 1$

$\Rightarrow \quad -\frac{7}{3} < x \le \frac{1}{3}$ [Dividing by 3]

$\therefore$ Solution set = $\left(-\frac{7}{3}, \frac{1}{3}\right]$

Solution set on number line is as shown by thick line in the figure

Illustration 13

Solve the inequation $-15 < \frac{3(x-2)}{5} \le 0$.

Given, $-15 < \frac{3(x-2)}{5} \le 0$.

$\Rightarrow \quad -75 < 3(x-2) \le 0$ [Multiplying by 5]

$\Rightarrow \quad -25 < x - 2 \le 0$ [Dividing by 3]

$\Rightarrow \quad -25 + 2 < x \le 0 + 2$ [Adding 2]

$\Rightarrow \quad -23 < x \le 2$

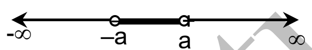
$\therefore$ Solution set = (–23, 2].

Solution set on number line is as show by thick line in the figure

Some Important Results

RESULT I If a is a positive real number, then

(i) $|x| < a \Leftrightarrow -a < x < a$ i.e. $x \in (-a, a)$



(ii) $|x| \le a \Leftrightarrow -a \le x \le a$ i.e. $x \in [-a, a]$

Proof (i) We know that:

$$|x| = \begin{cases} x, & \text{if } x \ge 0 \\ -x, & \text{if } x < 0 \end{cases}$$

So, we consider the following cases:

Case I When $x \ge 0$

In this case, $|x| = x$.

$\therefore \quad |x| < a \Rightarrow x < a$

Thus, in this case the solution set of the given inequation is given by

$x \ge 0$ and $x < a$

$\Rightarrow \quad 0 \le x < a$ ....(i)

Case II When $x < 0$:

In this case, $|x| = -x$

$\therefore \quad |x| < a \Rightarrow -x < a$

$\Rightarrow \quad x > -a$

Thus, in this case the solution set of the given inequation is given by

$x < 0$ and $x > -a$

$\Rightarrow \quad -a < x < 0$ ....(ii)

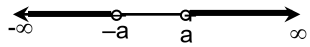
Combining (i) and (ii), we get

$|x| < a \Leftrightarrow -a < x < 0$ or $0 \le x < a \quad \Rightarrow |x| < a \Leftrightarrow -a < x < a$.

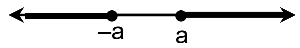
(ii) Proceeding as before, we get, $|x| \le a \Leftrightarrow -a \le x \le a$.

RESULT 2 If a is a positive real number, then

(i) $|x| > a \Leftrightarrow x < -a$ or $x > a$



(ii) $|x| \ge a \Leftrightarrow x \le -a$ or $x \ge a$



Proof (i) **Case I** When $x \ge 0$:

In this case,

$|x| = x$

$\therefore$ $|x| > a \Rightarrow x > a$

Thus, in this case the solution set of the inequation $|x| > a$ is given by

$x \ge 0$ and $x > a$

$\Rightarrow$ $x > a$ $[\because a > 0]$ …(i)

Case II When $x < 0$:

In this case, $|x| = -x$

$\therefore$ $|x| > a \Rightarrow -x > a$

$\Rightarrow x < -a$

Thus, in this case the solution set of the given inequation is given by

$x < 0$ and $x < -a \Rightarrow x < -a$ $[\because a > 0]$ …(ii)

Combining (i) and (ii), we get

$|x| > a \Leftrightarrow x < -a$ or $x > a$

(ii) Proceeding as before, we get

$|x| \ge a \Leftrightarrow x \le -a$ or $x \ge a$.

RESULT 3 Let r be a positive real number and a be a fixed real number. Then,

(i) $|x - a| < r \Leftrightarrow a - r < x < a + r$ i.e. $x \in (a - r, a + r)$

(ii) $|x - a| \le r \Leftrightarrow a - r \le x \le a + r$ i.e. $x \in [a - r, a + r]$

(iii) $|x - a| > r \Leftrightarrow x < a - r$, or $x > a + r$

(iv) $|x - a| \ge r \Leftrightarrow x \le a - r$, or $x \ge a + r$

Proof (i) Using Result 1 (i), we get

$|x - a| < r \Leftrightarrow -r < x - a < r$

$\Leftrightarrow a - r < x - a + a < a + r$

$\Leftrightarrow a - r < x < a + r$

(ii) Using result 1 (ii),

we have $|x - a| \le r \Leftrightarrow -r \le x - a \le r$

$\Leftrightarrow a - r \le x - a + a \le a + r$

$\Leftrightarrow a - r \le x \le a + r$

(iii) Using result 2(i), we have

$|x - a| > r \Leftrightarrow x - a < -r$, or $x - a > r$

$\Leftrightarrow x < a - r$, or $x > a + r$

(iv) Using result 2 (ii), we have,

$|x - a| \ge r \Leftrightarrow x - a \le -r$, or $x - a \ge r$

$\Leftrightarrow x \le a - r$, or $x \ge a + r$

Note: These results may be used directly for solving linear inequations involving absolute values.

RESULT 4 Let a, b be positive real numbers such that a < b. Then

(i) $a < |x| < b \Leftrightarrow x \in (-b, -a) \cup (a, b)$
(ii) $a \le |x| \le b \Leftrightarrow x \in [-b, -a] \cup [a, b]$
(iii) $a \le |x - c| \le b \Leftrightarrow x \in [-b + c, -a + c] \cup [a + c, b + c]$
(iv) $a < |x - c| < b \Leftrightarrow x \in (-b + c, -a + c) \cup (a + c, b + c)$

Proof (i) $a < |x| < b \Leftrightarrow |x| > a$ and $|x| < b$
$\Leftrightarrow (x < -a$ or $x > a)$ and $(-b < x < b)$
$\Leftrightarrow x \in (-b, -a) \cup (a, b)$
Similarly, we can prove other results.

Illustration 14

Solve: $|3x - 2| \le \frac{1}{2}$

We know that $|x - a| \le r \Leftrightarrow a - r \le x \le a + r$

$$\therefore \quad |3x - 2| \le \frac{1}{2} \Leftrightarrow 2 - \frac{1}{2} \le 3x \le 2 + \frac{1}{2}$$
$$\Leftrightarrow \frac{3}{2} \le 3x \le \frac{5}{2}$$
$$\Leftrightarrow \frac{1}{2} \le x \le \frac{5}{6}$$
$$\Leftrightarrow x \in [1/2, 5/6]$$

Hence, the solution set of the given inequation is the interval [1/2, 5/6].

Illustration 15

Solve: $|x - 2| \ge 5$

We know that:

$|x - a| \ge r \Leftrightarrow x \le a - r$, or $x \ge a + r$
$|x - 2| \ge 5 \Leftrightarrow x \le 2 - 5$, or $x \ge 2 + 5$
$\Leftrightarrow x \le -3$ or $x \ge 7$
$\Leftrightarrow x \in (-\infty, -3]$ or $x \in [7, \infty)$
$\Leftrightarrow x \in (-\infty, -3] \cup [7, \infty)$

Hence the solution set of the given inequation is $(-\infty, -3] \cup [7, \infty)$

Illustration 16

Solve: $1 \le |x - 2| \le 3$

We know that:
$a \le |x - c| \le b$.
$\Leftrightarrow \quad x \in [-b + c, -a + c] \cup [a + c, b + c]$
$\therefore \quad 1 \le |x - 2| \le 3 \Leftrightarrow x \in [-3 + 2, -1 + 2] \cup [1 + 2, 3 + 2]$
$\Leftrightarrow x \in [-1, 1] \cup [3, 5]$

Hence the solution set of the given inequation is $[-1, 1] \cup [3, 5]$.

Illustration 17

Solve the following system of inequations: $|x - 1| \le 5$, $|x| \ge 2$

The given system of inequations is

$|x - 1| \le 5$ …(i)
$|x| \ge 2$ …(ii)

Now, $|x-1| \le 5 \Rightarrow 1-5 \le x \le 1+5$ [$\because |x-a| \le r \Leftrightarrow a-r \le x \le a+r$]

$\Rightarrow \quad -4 \le x \le 6$

$\Rightarrow \quad x \in [-4, 6]$

Thus, the solution set of (i) is the interval $x \in [-4, 6]$

and $|x| \ge 2 \Rightarrow x \le -2$, or $x \ge 2$ [$\because \Rightarrow x \le -a$ or $x \ge a$]

Thus, the solution set of (ii) is $(-\infty, -2] \cup [2, \infty)$

The solution sets of inequations (i) and (ii) is $[-4, -2] \cup [2, 6]$

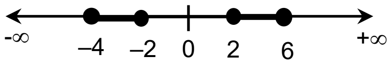


Illustration 18

Solve the inequation: $\left|\frac{2}{x-4}\right| > 1, \ x \ne 4$

We have, $\left|\frac{2}{x-4}\right| > 1, \ x \ne 4$

$\Rightarrow \quad \frac{2}{|x-4|} > 1$ $\left[\because \left|\frac{a}{b}\right| = \frac{|a|}{|b|}\right]$

$\Rightarrow \quad 2 > |x-4|$ [$\because |x-4| > 0$ for all $x \ne 4$]

$\Rightarrow \quad 4-2 < x < 4+2$ [$\because |x-a| < r \Leftrightarrow a-r < x < a+r$]

$\Rightarrow \quad 2 < x < 6 \quad \Rightarrow \quad x \in (2, 6)$

But, $x \ne 4$. Hence, the solution set of the given inequation is $(2, 4) \cup (4, 6)$

Illustration 19

Solve the inequation $\left|\frac{3x-4}{2}\right| \le \frac{5}{12}$

Given, $\left|\frac{3x-4}{2}\right| \le \frac{5}{12}$

$\Rightarrow \quad -\frac{5}{12} \le \frac{3x-4}{2} \le \frac{5}{12}$ [$\because |x| \le a \Leftrightarrow -a \le x \le a$]

$\Rightarrow \quad 12\left(-\frac{5}{12}\right) \le 12\left(\frac{3x-4}{2}\right) \le 12 . \frac{5}{12}$ [Multiplying by 12]

$\Rightarrow \quad -5 \le 18x - 24 \le 5$

$\Rightarrow \quad -5 + 24 \le 18x \le 5 + 24$ [Adding 24]

$\Rightarrow \quad 19 \le 18x \le 29$

$\Rightarrow \quad \frac{19}{18} \le x \le \frac{29}{18}$ [Dividing by 18]

$\therefore$ Solution set of given inequation $= \left[\frac{19}{18}, \frac{29}{18}\right]$

Solution set on number line is as shown in the figure by thick line.

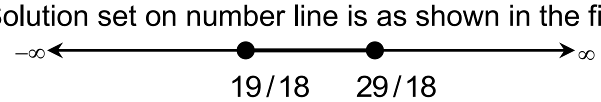


Illustration 20

Solve the inequation | 4 – x | + 1 < 3.

Given, | 4 – x | + 1 < 3

$\Rightarrow$ | 4 – x | < 2 [Subtracting 1]

$\Rightarrow$ –2 < 4 – x < 2 [$\because$ | x | < k $\Leftrightarrow$ – k < x < k]

$\Rightarrow$ –2 – 4 < –x < 2 – 4 [Subtracting 4]

$\Rightarrow$ –6 < – x < – 2

$\Rightarrow$ 6 > x > 2

$\Rightarrow$ 2 < x < 6 [Multiplying by –1]

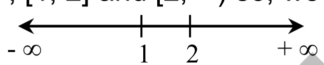
$\therefore$ Solution set = (2, 6)

$\therefore$ Solution set on the number line is as shown in the figure by thick line.

Illustration 21

Solve : |x – 1 | + | x – 2| ≥ 4

On the LHS of the given inequation there are two terms both containing modulus. By equating the expressions within the modulus to zero, we get x = 1, 2 as critical points. These points divide real line in three parts viz. (– ∞, 1] , [1, 2] and [2, ∞) so, we consider the following three cases.



Case I : When – ∞ < x < 1.

In this case, we | x – 1| = – (x – 1) and | x –2| = – (x–2)

$\therefore$ | x – 1| + | x –2| ≥ 4

$\Rightarrow$ –(x –1) – (x – 2) ≥ 4

$\Rightarrow$ – 2x + 3 ≥ 4

$\Rightarrow$ – 2x ≥ 1

$\Rightarrow$ $x \le -\frac{1}{2}$

But, – ∞ < x <1. Therefore, in this case the solution set of the given inequation is (–∞,– 1/2] …(i)

Case II : When 1 ≤ x < 2.

In this case, we have |x –1| = (x –1) and |x –2| = – (x –2)

$\therefore$ | x – 1| + |x – 2| ≥ 4

$\Rightarrow$ x –1 – (x – 2) ≥ 4

$\Rightarrow$ 1 ≥ 4, which is an absurd result.

So, the given inequation has no solution for x ∈[1,2).

Case III : When x ≥ 2

In this case, we have

| x –1 | = x – 1 and | x – 2 | = x – 2

$\therefore$ | x – 1| + | x – 2 | ≥ 4

$\Rightarrow$ x – 1 + x – 2 ≥ 4

$\Rightarrow$ 2x – 3 ≥ 4

$\Rightarrow$ 2x ≥ 7

$\Rightarrow \quad x \geq \dfrac{7}{2}$

But, $x \geq 2$. Therefore, in this case the solution set of the given inequation is $[7/2, \infty)$ …(ii)

Combining (i) and (ii), we obtain that the solution set of the given inequation is $(-\infty, -1/2] \cup [7/2, \infty)$

Illustration 22

Solve : $\dfrac{|x-1|}{x+2} < 1$

We have, $\dfrac{|x-1|}{x+2} < 1$

$\Rightarrow \quad \dfrac{|x-1|}{x+2} - 1 < 0$

$\Rightarrow \quad \dfrac{|x-1| - (x+2)}{x+2} < 0$

Now the following cases arise.

Case I : When $x - 1 \geq 0$ i.e. $x \geq 1$.

In this case, we have $|x-1| = x - 1$

$\therefore \quad \dfrac{|x-1| - (x+2)}{x+2} < 0 \quad \Rightarrow \quad \dfrac{(x-1)-(x-2)}{x+2} < 0$

$\Rightarrow \quad \dfrac{-3}{x+2} < 0$

$\Rightarrow \quad x + 2 > 0 \qquad \left[\because \dfrac{a}{b} < 0 \text{ and } a < 0 \Rightarrow b > 0\right]$

$\Rightarrow \quad x > -2$

But, $x \geq 1$. Therefore, $x \geq 1$.

Thus, in this case the solution set of the given inequation is $[1, \infty)$ ….(i)

Case II : When $x - 1 < 0$ i.e. $x < 1$:

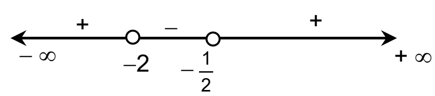
In this case, we have $|x-1| = -(x-1)$.

$\therefore \quad \dfrac{|x-1| - (x+2)}{x+2} < 0$

$\Rightarrow \quad \dfrac{-(x-1)-(x+2)}{x+2} < 0$

$\Rightarrow \quad -\dfrac{2x+1}{x+2} < 0$

$\Rightarrow \quad \dfrac{2x+1}{x+2} > 0$



$\Rightarrow \quad x \in (-\infty\ -2) \cup (-1/2, \infty)$.

But, $x < 1$. Therefore $x \in (-\infty, -2) \cup (-1/2, 1)$

Thus, in this case the solution set of the given inequation is $(-\infty, -2) \cup (-1/2, 1)$

Combining (i) and (ii), we obtain that the solution set of the given inequation as $(-\infty, -2) \cup (-1/2, \infty)$.

Practice Assignment– I

Solve the following linear inequalities and represent the solution on a number line

1. $24x < 100$, when :

(i) $x \in N$ (ii) $x \in Z$ (iii) $x \in R$

2. $5x - 8 \leq 3x$, when

(i) $x \in N$ (ii) $x \in Z$ (iii) $x \in R$

3. $3x - 4 > 7 - 2x$

4. $\frac{x+1}{2} \geq \frac{2-x}{-3}$

5. $\frac{5x-7}{11} > \frac{8-x}{-5}$

6. $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$

7. $\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x-6)$

8. $\frac{3(x-2)}{5} < \frac{5(2-x)}{3}$

9. $\frac{x}{x-4} > 0$

10. $\frac{x+1}{x-1} > 4$

11. $\frac{3x-1}{x+7} \leq 5$

12. $\frac{6x-1}{1-x} < 8$

13. $|x| \leq 4$

14. $|x| \geq 7$

15. $|3x - 4| \leq -6$

16. $|1 - 2x| \leq 11$

17. $\frac{|x-2|}{x-2} > 0$

18. $|x + 2| > |3x - 5|$

19. $\frac{|x+2| - x}{x} < 2$

20. $|x-1| + |x - 2| + |x - 3| \geq 6$

21. $\frac{|x+3| + x}{x+2} > 1$

22. The marks obtained by a student in two tests were 70 and 75. Find the number of minimum marks he should get in the third test to have an average of at least 60 marks.

23. To be placed under first division, one must obtain an average of 60 marks or more. If the marks obtained by a student in four examinations be 46, 84, 54, 79, find the minimum marks that the student must obtain in fifth examination so that he may obtain first division.

24. The longest side of a triangle is twice the shortest side and the third side is 3 cm longer than the shortest side. If the perimeter of the triangle is at least 39 cm, find the minimum length of the longest side.

25. An electrician can be paid under two schemes as given below:
I: Rs. 500 and Rs. 70 per hour II: Rs. 120 per hour.
If the job takes x hours, for what values of x does the (i) Scheme I (ii) Scheme II give the electrician the better wages.

Solutions of System of Linear Inequations in one Variable

We adopt the following algorithmic approach to solve the system of linear inequations in one variable:

Algorithmic Approach

(i) Get the system of linear inequations.
(ii) Solve each inequation and get the solution sets. Also represent them on real line.
(iii) Obtain the intersection of the solution sets obtained in step (ii), by drawing the graphs of the solution sets.
The set, which is obtained in step (iii), is the reqd. solution set.

Illustration 23

Let A be the solution set of inequation $8x - 1 > 5x + 2$ and B be the solution set of inequation $7x - 2 \geq 3(x + 6)$, where $x \in N$. Find the set $A \cap B$.

Given, $8x - 1 > 5x + 2$

$\Rightarrow$ $8x - 5x > 1 + 2$ [Transferring 5x on L.H.S. and 1 on R.H.S.]

$\Rightarrow$ $3x > 3$

$\Rightarrow$ $x > 1$ [Dividing by 3]

$\therefore$ $A = \{2, 3, 4, 5, \ldots\}$ [$\because x \in N$]

Again, $7x - 2 \geq 3(x + 6)$ $\Rightarrow$ $7x - 2 \geq 3x + 18$.

$\Rightarrow$ $7x - 3x \geq 2 + 18$ [Transferring 3x on L.H.S. and –2 on R.H.S.]

$\Rightarrow$ $4x \geq 20 \Rightarrow x \geq 5$

$\therefore$ $B = \{5, 6, 7, 8, \ldots\}$

$\therefore$ $A \cap B = \{5, 6, 7, 8, \ldots\}$. [$\because x \in N$]

$\therefore$

Illustration 24

Solve the following system of inequations $2x - 3 < 7$, $2x > -4$.

Given, $2x - 3 < 7$ ....(1)

and $2x > -4$ ....(2)

Now, $2x - 3 < 7 \Rightarrow 2x < 10$ [Adding 3 to both sides]

$\Rightarrow$ $x < 5$ [Dividing by 2]

$\Rightarrow$ $-\infty < x < 5$ ......(3)

Again $2x > -4$ $\Rightarrow x > -2$ [Dividing by 2]

$\Rightarrow$ $-2 < x < \infty$ ....(4)

From (3) and (4), common values of x are given by $-2 < x < 5$ ....(5)

$\therefore$ Solution set = $(-2, 5)$.

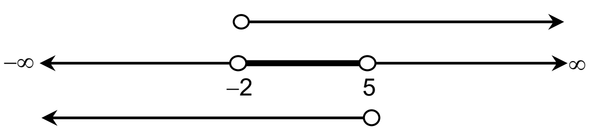


Illustration 25

Solve the following system of inequations $5x - 7 < 3(x + 3)$, $1 - \frac{3x}{2} \le x - 4$.

Given, $5x - 7 < 3(x + 3)$ ……..(1)

and $1 - \frac{3x}{2} \le x - 4$. ……..(2)

Now $5x - 7 < 3(x + 3)$

$\Rightarrow$ $5x - 7 < 3x + 9$

$\Rightarrow$ $5x - 3x < 7 + 9$ [Transferring 3x on L.H.S. and –7 on R.H.S.]

$\Rightarrow$ $2x < 16$ $\Rightarrow$ $x < 8$

$\Rightarrow$ $-\infty < x < 8$ ……..(3)

Again, $1 - \frac{3x}{2} \le x - 4$

$\Rightarrow$ $2 - 3x \le 2x - 8$ [Multiplying by 2]

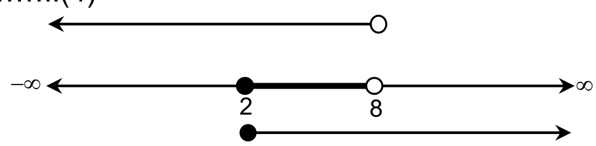
$\Rightarrow$ $-3x - 2x \le -2 - 8$ Transferring 2x on L.H.S. and 2 on R.H.S.]

$\Rightarrow$ $-5x \le -10$

$\Rightarrow$ $x \ge 2$ [Dividing by –5]

$\Rightarrow$ $2 \le x < \infty$ ……..(4)

From (3) and (4), common values of x are given by $2 \le x < 8$



∴ Solution set = [2, 8).

Illustration 26

Solve the following system of inequations:

$2x - 7 > 5 - x$, $11 - 5x \le 1$.

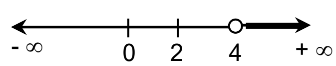
The given inequations are $2x - 7 > 5 - x$ ….(1)

and $11 - 5x \le 1$ ….(2)

From (1), $2x + x > 7 + 5 \Rightarrow 3x > 12$

$\Rightarrow$ $x > 4$.

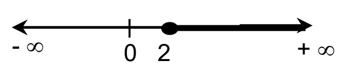
Thus the solution set of (1) is $(4, \infty)$.



From (2), $11 - 5x \le 1$

$\Rightarrow$ $-5x \le 1 - 11$

$\Rightarrow$ $-5x \le -10$ $\Rightarrow$ $x \ge 2$.



Thus the solution set of (2) is $[2, \infty)$.

Clearly the intersection of these solution sets is the sets $(4, \infty)$.

Hence the solution set of the given system of inequations is $(4, \infty)$.

Illustration 27

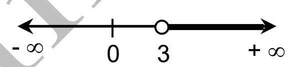
Solve the following system of inequations:

$$\frac{5x}{4}+\frac{3x}{8}>\frac{39}{8} \text{ and } \frac{2x-1}{12}-\frac{x-11}{3}<\frac{3x+1}{4}.$$

The given inequations are $\frac{5x}{4}+\frac{3x}{8}>\frac{39}{8}$ and $\frac{2x-1}{12}-\frac{x-11}{3}<\frac{3x+1}{4}$

From(1), $\frac{10x+3x}{8}>\frac{39}{8} \Rightarrow 13x>39 \qquad \Rightarrow \qquad x > 3.$

Thus the solution set of (1) is $(3, \infty)$.



From (2), $\frac{(2x-1)-4(x-11)}{12}<\frac{3x+1}{4}$

$\Rightarrow \qquad \frac{-2x+43}{12}<\frac{3x+1}{4}$

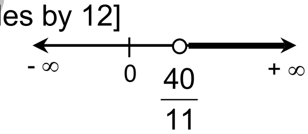
$\Rightarrow \qquad -2x + 43 < 3(3x+1)$ [Multiplying both sides by 12]

$\Rightarrow \qquad -2x + 43 < 9x + 3$

$\Rightarrow \qquad -2x - 9x < 3 - 43$

$\Rightarrow \qquad -11x < -40$

$\Rightarrow \qquad x > \frac{40}{11}.$



Thus the solution set is $\left(\frac{40}{11},\infty\right)$.

Clearly the intersection of these solutions sets is the set $\left(\frac{40}{11},\infty\right)$.

Hence the solution set of the given system of inequations is $\left(\frac{40}{11},\infty\right)$.

Illustration 28

Solve the following system of inequations:

$$2(2x+3)-10<6(x-2) \text{ and } \frac{2x-3}{4}+6\geq 2+\frac{4x}{3}.$$

The given inequations are $2(2x + 3) - 10 < 6(x - 2)$ and $\frac{2x-3}{4}+6\geq 2+\frac{4x}{3}$

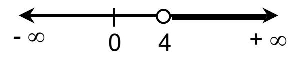
From (1), $\qquad 4x + 6 - 10 < 6x - 12$

$\Rightarrow \qquad 4x - 4 < 6x - 12$

$\Rightarrow \qquad 4x - 6x < 4 - 12$

$\Rightarrow \qquad -2x < -8$

$\Rightarrow \quad x > 4.$



Thus the solution set of (1) is $(4,\infty)$.

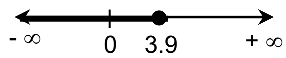
From (2), $\frac{2x-3+24}{4} \geq \frac{6+4x}{3}$

$\Rightarrow \quad \frac{2x+21}{4} \geq \frac{4x+6}{3} \quad \Rightarrow \quad 3(2x + 21) \geq 4\,(4x + 6)$

$\Rightarrow \quad 6x + 63 \geq 16x + 24 \quad \Rightarrow \quad 6x - 16x \geq 24 - 63$

$\Rightarrow \quad -10x \geq -39$

$\Rightarrow \quad x \leq \frac{39}{10} \quad x \leq 3.9.$



$\Rightarrow$

Thus the solution set of (2) is $(-\infty, 3.9]$.

Clearly the intersection of these solution sets is the set ϕ.

Hence the given system of inequations has no solution.

Illustration 29

Solve the following system of linear inequalities:

$3x + 9 \leq 0, \;\; 7x - 2 < 0, \; 1 - x > 9.$

The given inequalities are

$3x + 9 \leq 0$ ...(1) $\quad 7x - 2 < 0$ ...(2) $\quad 1 - x > 9$ ...(3)

(1) $\quad 3x \leq -9 \Rightarrow x \leq -\frac{9}{3} \quad \Rightarrow \quad x \leq -3 \quad \therefore \quad x \in (-\infty, -3]$

(2) $\quad 7x < 2 \quad \Rightarrow \quad x < \frac{2}{7} \quad \therefore \quad x \in \left(-\infty, \frac{2}{7}\right)$

(3) $\quad -x > 8 \quad \Rightarrow \quad x < -8 \quad \therefore \quad x \in (-\infty, -8)$

$\therefore$ Solution set of given system

$= (-\infty, -3] \cap \left(-\infty, \frac{2}{7}\right) \cap (-\infty, -8) = (-\infty, -8).$

This solution set can also be shown on the number line as follows:

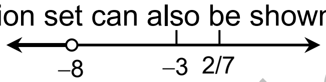


Illustration 30

Solve the following system of linear inequalities:

$2x - 7 \geq 0, \; \frac{x-4}{x+4} > 1.$

The given inequalities are

$2x - 7 \geq 0$ ...(1)

and $\frac{x-4}{x+4} > 1$ ...(2)

(1) $\quad 2x \geq 7 \quad \Rightarrow \quad x \geq 3.5. \quad \therefore \quad x \in [3.5, \infty).$

(2) $\quad \frac{x-4}{x+4} - 1 > 0 \quad \Rightarrow \quad \frac{(x-4)-(x+4)}{x+4} > 0 \quad \Rightarrow \quad -\frac{8}{x+4} > 0$

$\Rightarrow \quad x + 4 < 0 \quad \Rightarrow \quad x < -4 \quad \therefore \quad x \in (-\infty, -4)$

$\therefore$ Solution set of given system $= [3.5, \infty) \cap (-\infty, -4) = \phi.$

The solution set can also be shown on the number line as follows:

Illustration 31

Solve the following system of inequalities:

$|2x + 3| \le 4, \quad |x - 4| \ge 7$.

The given inequalities are

$|2x + 3| \le 4$ …(1)

and $|x - 4| \ge 7$ …(2)

(1) $-4 \le 2x + 3 \le 4$ [Using $|x| \le k \Rightarrow -k \le x \le k$]

$-4 - 3 \le 2x + 3 - 3 \le 4 - 3$

$-7 \le 2x \le 1$

$\Rightarrow \quad -\frac{7}{2} \le x \le \frac{1}{2}$ $\left(\because \frac{1}{2} > 0\right)$

$\therefore \quad x \in \left[-\frac{7}{2}, \frac{1}{2}\right]$

(2) $x - 4 \le -7$

or $x - 4 \ge 7$ [Using $|x| \ge k \Rightarrow x \le -k$ or $x \ge k$]

$\Rightarrow$ $x - 4 + 4 \le -7 + 4$ or $x - 4 + 4 \ge 7 + 4$

$\Rightarrow$ $x \le -3$ or $x \ge 11$

$\therefore$ $x \in (-\infty, -3] \cup [11, \infty]$

$\therefore$ Solution set of given system

$= \left[-\frac{7}{2}, \frac{1}{2}\right] \cap ((-\infty, -3] \cup [11, \infty))$

$= \left(\left[-\frac{7}{2}, \frac{1}{2}\right] \cap (-\infty, -3]\right) \cup \left(\left[-\frac{7}{2}, \frac{1}{2}\right] \cap [11, \infty)\right) = \left[-\frac{7}{2}, -3\right] \cup \phi = \left[-\frac{7}{2}, -3\right]$.

The solution set can also be shown on the number line as follows:

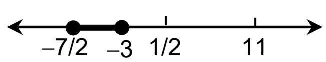


Illustration 32

IQ of a person is given by the formula IQ = $\frac{MA}{CA} \times 100$, where MA is mental age and CA is chronological age. If $80 \le IQ \le 140$ for a group of 12 year children, find the range of their mental age.

We have $80 \le IQ \le 140$, CA = 12.

The formula is $IQ = \frac{MA}{CA} \times 100$.

$$IQ = \frac{MA}{12} \times 100 \text{ or } IQ = \frac{25MA}{3}$$

$$80 \le IQ \le 140$$

$\Rightarrow$ $80 \le \frac{25MA}{3} \le 140$

$\Rightarrow$ $3(80) \le 25\,MA \le 3(140)$

$\Rightarrow$ $240 \le 25\,MA \le 420$ $\Rightarrow$ $\frac{240}{25} \le MA \le \frac{420}{25}$

$\Rightarrow$ $9.6 \le MA \le 16.8$.

Illustration 33

Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

Let x and x + 2 be the required consecutive odd positive integers.

$\therefore$ By the given conditions,

$x < 10$ …(1)

$x + 2 < 10$ …(2)

$x + (x + 2) > 11$ …(3)

(2) $x < 8$

(3) $2x + 2 > 11$

$\Rightarrow$ $2x > 9$

$\Rightarrow$ $x > \frac{9}{2}$

$\therefore$ $x < 10, x < 8, x > 4\frac{1}{2}$ $\therefore$ Possible values of x are 5,7.

$\therefore$ Possible pairs are (5,5 + 2), (7, 7 + 2) or (5, 7), (7, 9).

Illustration 34

Find all pairs of consecutive even positive integers both of which are larger than 5, such that their sum is less than 23.

Let x and x + 2 be the required consecutive even positive integers.

$\therefore$ By the given conditions,

$x > 5$ …(1)

$x + 2 > 5$ …(2)

$x + (x + 2) < 23$ …(3)

(2) $x > 3$ …(4)

(3) $2x + 2 < 23$

$\Rightarrow$ $2x < 21$

$\Rightarrow$ $x < 10.5$ …(5)

(1), (4) and (5)

$\Rightarrow$ $5 < x < 10.5$

$\Rightarrow x = 6, 8, 10$ ($\because$ x is even integer)

$\therefore$ Possible pairs are (6, 8), (8,10), (10, 12).

Illustration 35

A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths for the shortest board if the third piece is to be at least 5 cm longer than the second?

Let the lengths of pieces of board be x cm, y cm, z cm and $x < y < z$. By the given conditions,

$x + y + z \leq 91$ ...(1)

$y = x + 3$ ...(2)

$z = 2x$ ...(3)

$z \geq y + 5$ ...(4)

∴ (1) $x + (x + 3) + 2x \leq 91$

$\Rightarrow 4x + 3 \leq 91$

$4x \leq 88 \Rightarrow x \leq 22$

(4) $2x \geq (x + 3) + 5 \Rightarrow x \geq 8$

∴ The length of the shortest board lies between 8 cm and 22 cm both inclusive.

Practice Assignment II

Solve the following systems of linear inequalities.

1. $3x - 7 < 5 + x,\ 11 - 5x \le 1.$

2. $x - 3 > 0,\ 2x + 9 > 5,\ 3x + 4 < -5$

3. $\frac{x-4}{7} < 3, \frac{2x+5}{-3} > 4$

4. $\frac{1-7x}{2} > 3, \frac{3x+8}{5} < -11$

5. $3x + 9 < -x + 19,\ 2x - 5 < 1 - x$

6. $x - 5 > 0,\ \frac{2x-4}{x+2} < 2$

7. $\frac{4x+5}{2x-7} < 2, \frac{8x-5}{2x+11} \ge 4$

8. $\frac{x+7}{x-8} > 2, \frac{2x+1}{7x-1} > 5$

9. $| x - 4 | < 5,\ | 2x + 5 | > 7$

10. $| 4 - 3x | > 15,\ |15x - 7| < 7.$

11. Find all pairs of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less than 40.

12. Find all pairs of consecutive even natural numbers, both of which are smaller than 15, such that their sum is not less than 22.

13. A solution is kept between 68°F and 77°F. What is the range of temperature in degree Celsius(C) if the conversion formula is given by $F = \frac{9}{5}C + 32$, where C and F represent temperature in degree Celsius and degree Fahrenheit respectively?

14. A manufacturer has 600 litres of a 12 % solution of acid. How many litres of a 30 % acid solution must be added to it so that acid content in the resulting mixture will be more than15 % but less than 18%?

15. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

16. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added ?

Graphical solutions of Linear inequations in two variables

If a, b, c are real numbers, then ax + by < c, ax + by ≤ c, ax + by > c, ax + by ≥ c are called linear inequations in two variables x and y. We know that the graph of ax + by = c is a straight line, which divides the xy-plane into two parts which are represented by ax + by ≤ c and ax + by ≥ c. These are known as **closed half-spaces**. And the regions represented by ax + by < c and ax + by > c are called **open half-spaces**.

Algorithmic Approach

(i) Convert the given inequation; say ax + by ≤ c into the equation ax + by = c, which is a st. line in xy-plane.

(ii) Find the points where ax + by = c meets the x-axis (y = 0) and y-axis (x =0).

(iii) Join the two points, as obtained in step (ii) and obtain the graph of the st. line ax + by = c.

(iv) (a) If the inequality is ax + by < c, draw the dotted line.
(b) If the inequality is ax + by ≤ c, draw the thick line.

(v) Select the point (0, 0), if it does not lie on the line. Substitute the co-ordinates of the point in the given inequation. If the inequation is satisfied, shade the portion of the plane in which the point lies, otherwise shade the portion of the plane in which the point does not lie.

Remark. If the inequalities are ax + by ≤ c and ax + by ≥ c, the points on the line are in the shaded region while in the case of inequalities ax + by < c and ax + by > c, the points on the line are not in the shaded region.

Illustration 36

Solve x > – 2 graphically in XY-plane.

Given inequation is x > –2 ….(1)

Its corresponding equation is x = – 2 ….(2)

Line (2) is parallel to y-axis and cuts x-axis at (–2, 0).

O(0, 0) does not lie on lie (2).

Also for O(0, 0), inequation x > – 2 is satisfied. Line (2) divide the XY-plane in two half-planes. The given inequation represents the open half-plane made by the line (2) which contains the origin.

Solution set = set of all points in the shaded region excluding the line (2).

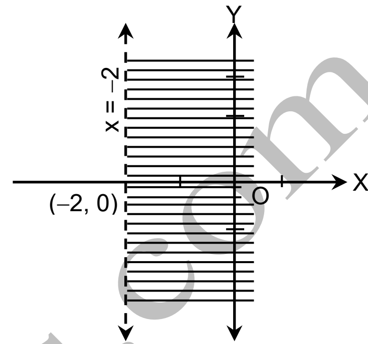


Illustration 37

Solve the following inequation graphically 3y –5x < 30

Given inequation is 3y – 5x < 30 …..(1)

Corresponding equation is 3y – 5x = 30 …..(2)

y = 0 ⇒ – 5x = 30 ⇒ x = –6.

Hence line (2) cuts x-axis at (–6, 0).

Again, x = 0 ⇒ 3y = 30 ⇒ y = 10.

∴ Line (2) cuts y-axis at (0, 10). The point A(–6, 0) and B(0, 10) lies on line (2).

The line joining A and B divides the XY-plane in two half-planes. Origin O(0, 0) does not lie on line (2).

Also for O(0, 0) inequation 3y – 5x < 30 is satisfied. Therefore, the inequation (1) represents the open half plane made by line AB which contains the origin.

Solution set is set of all points in the shaded region excluding points on line AB.

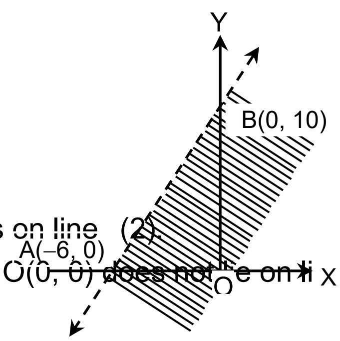


Illustration 38

Draw the graphs of the following inequalities:

(i) x + 2y ≤ 10 (ii) 2x – y > 5.

(i) The given inequality is x + 2y ≤ 10. …(1)

The corresponding equation is x + 2y = 10. …(2)

On the line x + 2y = 10,

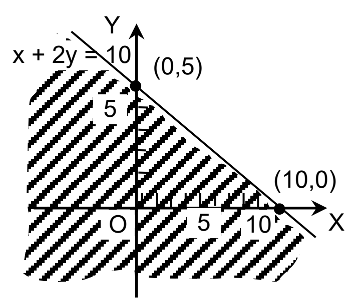
x = 0 ⇒ 0 + 2y = 10 ⇒ y = 5

y = 0 ⇒ x + 2(0)=10 ⇒ x = 10

∴ The points (0, 5) and (10, 0) are on the line (2).

The origin does not lie on (2) and it lies in the half plane of (1), if 0 + 2(0) ≤ 10, which is true.

∴The closed half-plane containing the origin is the graph of the given inequality.



(ii) The given inequality is 2x – y > 5. …(1)

The corresponding equation is 2x – y = 5. …(2)

On the line 2x – y = 5,

x = 0

⇒ 2(0) – y = 5

⇒ y = –5

y = 0

⇒ 2x – 0 = 5

⇒ x = 5/2

∴ (0, –5) and (5/2, 0) are on the line (2).

The origin does not lie on (2) and it lies in the half plane(1), if 2(0) – 0 > 5, which is not true.

∴ The open half-plane not containing the origin is the graph of the given inequality.

In this case, the points lying on the line 2x – y = 5 do not constitute part of the graph of given inequation 2x – y > 5.

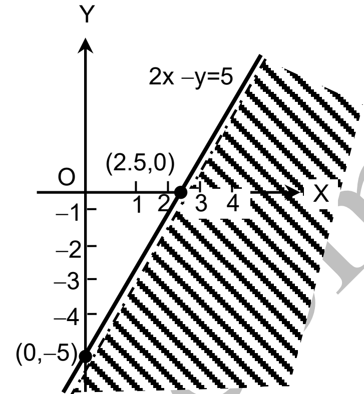


Illustration 39

Draw the graph of the following inequality: x + y ≤ 0

The given inequality is x + y ≤ 0. …(1)

The corresponding equation is x + y =0. …(2)

On the line x + y =0.

x = 0 ⇒ 0 + y = 0 ⇒ y = 0

x = 2 ⇒ 2 + y = 0 ⇒ y = – 2.

∴ (0, 0) and (2, –2)are on the line (2)

The point (2,4) is not on the line and it lies in the half place of (1)if 2 + 4 ≤ 0, which is not true.

∴ The closed half plane not containing the point (2,4) is the graph of the given inequality.

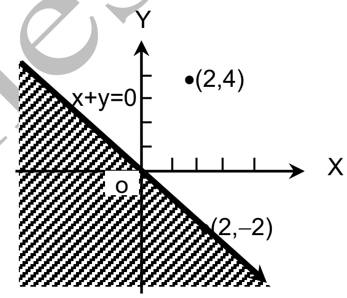


Illustration 40

Draw the graph of the following inequality 3y – 5x < 30

The given inequality is 3y – 5x < 30.

⇒ –5x + 3y – 30 < 0

⇒ –(–5x + 3y– 30) > – 0

⇒ 5x – 3y + 30 > 0 …(1)

The corresponding line is

5x – 3y + 30 = 0 …(2)

On the line 5x – 3y + 30 = 0,

x = 0

⇒ 0 – 3y + 30 = 0

⇒ y = 10

y = 0

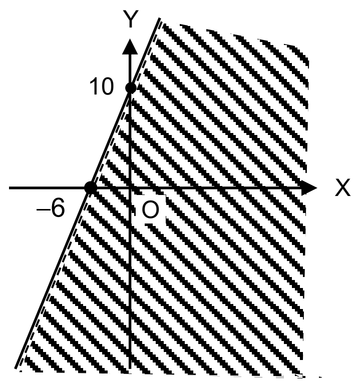
⇒ 5x – 0 + 30 = 0

⇒ x = –6.

∴ (0,10) and (–6,0) are on the line (2).

The origin does not lie on (2) and it lies in the half-plane of (1) if 5(0) – 3(0) + 30 > 0, which is true.

The open half-plane containing the origin is the graph of the given inequality.



Solution of System of Linear Inequalities in two Variables and the graph of its solution set

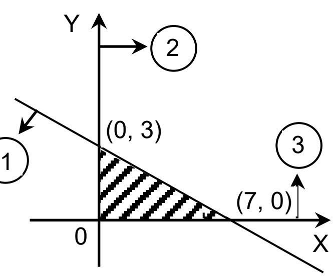
Two or more linear inequalities in two variables constitute a system of linear inequalities in two variables.
We know that the solution set of a linear inequality in two variables is the set of points in the half-plane represented by the corresponding linear inequality.
The solution set of a system of linear inequalities in two variables is defined as the intersection of the solution sets of the linear inequalities in the system.
The solution set of a system of linear inequalities may even be the empty set. For example, there is no point in the X-Y plane which may satisfy the inequalities $x + 2y < 2$ and $x + 2y \geq 2$.
A linear equation is also called a linear constraint because it restricts our freedom of choice of the values of x and y.
Thus, the solution set of a system of linear constraints (i.e., inequalities) is the intersection of the solution sets of linear constraints in the system. The solution set of a system of linear constraints is empty or a polygon, i.e., a region bounded by straight lines or an unbounded region with straight-line boundaries.

Working rules for drawing the graph of a system of Linear inequalities in two variables

Step I. Draw straight lines corresponding to each linear inequality in the given system. Also mark each line as to which side of the line is the required half-plane.
Step II. $x \geq 0$ is the closed half-plane on the right of y-axis.
Step III. $y \geq 0$ is the closed half-plane above the x-axis.
Step IV. Shade the region, which is common to all half-planes. This shaded region represents the graph of the given system of linear inequalities.

Illustration 41

Graphically solve the system of linear inequalities:
$3x + 7y - 21 \leq 0$, $x \geq 0$; $y \geq 0$.

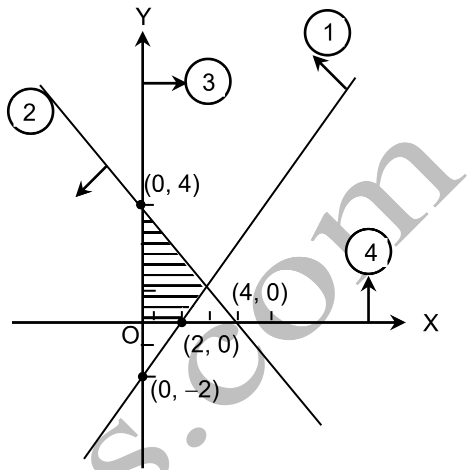


The given system is $3x + 7y - 21 \leq 0$ …(1)
$x \geq 0$ …(2)
$y \geq 0$ …(3)
The line corresponding to (1) is $3x + 7y - 21 = 0$ …(4)
On the line (4),
$x = 0$
$\Rightarrow$ $0 + 7y - 21 = 0$
$\Rightarrow$ $y = 3$, $y = 0$
$\Rightarrow$ $3x + 0 - 21 = 0$
$\Rightarrow$ $x = 7$.
$\therefore$ (0,3) and (7,0) are on the line (4). (0,0) is not on the line (4) and it lies in the half-plane of (1) if $3(0) + 7(0) - 21 \leq 0$, which is true.
$\therefore$ The closed half-plane containing the origin is the graph of the inequality (1). The inequality $x \geq 0$ represent the closed half-plane on the right of Y-axis.
The inequality $y \geq 0$ represents the closed half-plane above the X-axis.
The graph of the given system is the intersection (shown by shaded lines) of half-planes of the inequalities in the system.

Illustration 42

Graphically solve the system of linear inequalities:

$x - y \le 2$, $x + y \le 4$, $x \ge 0$, $y \ge 0$.



The given system of linear inequalities is

$x - y \le 2$ …(1)

$x + y \le 4$ …(2)

$x \ge 0$ …(3)

$y \ge 0$ …(4)

The line corresponding to (1) is

$x - y = 2$ …(5)

On the line (5),

$x = 0 \Rightarrow 0 - y = 2 \Rightarrow y = -2$

$y = 0 \Rightarrow x - 0 = 2 \Rightarrow x = 2.$

∴ (0, –2) and (2, 0) are on the line (5). (0, 0) is not on this line and it lies in the half-plane of (1) if $0 - 0 \le 2$, which is true

∴ The closed half-plane containing the origin is the graph of the inequality (1).

The line corresponding to (2) is $x + y = 4$ …(6)

On the line (6),

$x = 0 \Rightarrow 0 + y = 4 \Rightarrow y = 4$ and $y = 0 \Rightarrow x + 0 = 4 \Rightarrow x = 4.$

∴ (0, 4) and (4,0) are on the line (6). (0,0) is not on this line and it lies in the half-plane of (2) if $0 + 0 \le 4$, which is true.

∴ The closed half-plane containing the origin is the graph of the inequality (2).

The inequality $x \ge 0$ represents the closed half-plane on the right of Y-axis.

The inequality $y \ge 0$ represents the closed half-plane above X-axis.

The graph of the given system is the intersection of half-planes of the inequalities in the system.

The shaded region is the required graph of the given system of inequalities.

Illustration 43

Graphically solve the system of linear inequalities:

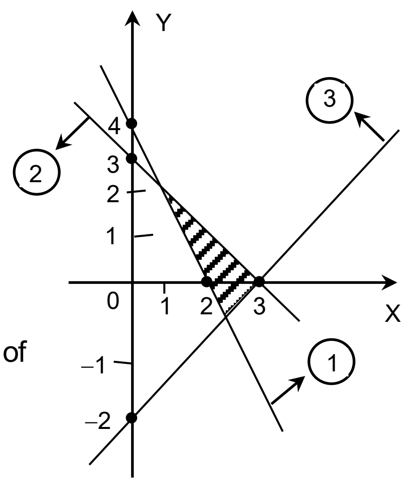
$2x + y \ge 4$, $x + y \le 3$, $2x - 3y \le 6$.

The linear inequalities in the given system are:

$2x + y \ge 4$ …(1)

$x + y \le 3$ …(2)

$2x - 3y \le 6$ …(3)



The line corresponding to (1) is $2x + y = 4$ …(4)

(0, 4) and (2, 0) are on (4). (0, 0) is not on (4) and it lies in the half-plane of (1), if $2(0) + 0 \ge 4$, which is not true.

∴ The closed half-plane not containing the origin is the graph of (1).

The line corresponding to (2) is $x + y = 3$. …(5)

(0, 3) and (3, 0) are on (5). (0, 0) is not on (5) and it lies in the half-plane of (2) if $0 + 0 \le 3$, which is true.

∴ The closed half-plane not containing the origin is the graph of (2).

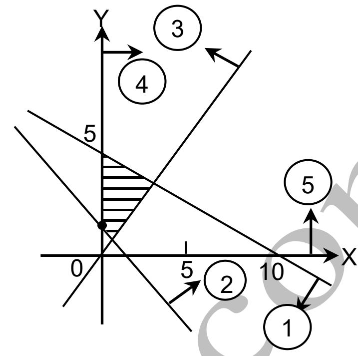
The line corresponding to (3) is $2x - 3y = 6$. …(6)

(0, –2) and (3, 0) are on (6). (0, 0) is not on (6) and it lies in the half-plane of (3), if $2(0) - 3(0) \le 6$, which is true.

∴ The closed half-plane containing the origin is the graph of (3).
The shaded region is the required graph of the given system of inequalities.

Illustration 44

Graphically solve the system of linear inequalities:
$x + 2y \le 10$, $x + y \ge 1$, $x - y \le 0$, $x \ge 0$, $y \ge 0$.



The linear inequalities in the given system are:

$x + 2y \le 10$ …(1)
$x + y \ge 1$ …(2)
$x - y \le 0$ …(3)
$x \ge 0$ …(4)
$y \ge 0$ …(5)

The line corresponding to (1) is
$x + 2y = 10$ …(6)

(0, 5) and (10, 0) are on (6). (0, 0) is not on (6) and it lies in the half-plane of (1), if $0 + 2(0) \le 10$, which is true.

∴ The closed half-plane containing the origin is the graph of (1).
The line corresponding to (2) is $x + y = 1$ …(7)
(0, 1) and (1, 0) are on (7). (0, 0) not on (7) and it lies in the half-plane of (2) if $0 + 0 \ge 1$, which is not true.

∴ The closed half-plane not containing the origin is the graph of (2).
The line corresponding to (3) is $x - y = 0$ …(8)
(0, 0) and (5, 5) are on (8). (4, 0) is not on (8) and it lies in the half-plane of (3) if $4 - 0 \le 0$, which is not true.

∴ The closed half-plane not containing (4, 0) is the graph of (3).
The inequalities $x \ge 0$, and $y \ge 0$, represent the closed half-planes on the right of y-axis and above X-axis respectively.
The shaded region is the required graph of the given system of inequalities.

Practice Assignment III

Draw the graphs of the following system of inequalities and find the solution set

1. $3x - 4y \le 12$
2. $\frac{x}{9} + \frac{y}{6} \ge 1$
3. $y + 8 \ge 2x$
4. $2x + y \ge 8$
   $x + 2y \ge 10$
5. $x + y \ge 1$
   $3x + y \le 4$
   $x , y \ge 0$
6. $x - y \ge 0$
   $3x - y + 3 \le 0$
   $x, y \ge 0$
7. $2x + 3y \le 6$
   $x + 4y \le 4$
   $x , y \ge 0$
8. $3x + 4y \le 60$
   $x + 3y \le 30$
   $x \ge 0, y \ge 0$

9. $4x + 3y \le 60$
$y \ge 2x$
$x \ge 3$
$x \ge 0, y \ge 0$

10. $3x + 2y \le 150$
$x + 4y \le 80$
$x \le 15, x, y \ge 0$

11. $x + y \ge 1$
$7x + 9y \le 63$
$x \le 6, y \le 5$
$x, y \ge 0$

12. $3y - x \le 10$
$x + y \le 6$
$x - y \le 2$
$x, y \ge 0$

13. $x + y \le 4$
$3x + y \ge 4$
$x + 5y \ge 4$
$x \le 3, y \le 3$
$x, y \ge 0$

14. $x + 2y \le 3$, $3x + 4y \ge 12$, $x \ge 0$, $y \ge 1$

15. Verify that the solution set of following linear inequalities is empty: $x - 2y \ge 0$, $2x - y \le -2$, $x \ge 0$, $y \ge 0$.

Objective Assignment

1. If x, y, z are real numbers, which of the following is not correct
(a) $x > z$ and $y > 0 \Rightarrow xy > yz$
(b) $x > y \Rightarrow x + z > y + z$
(c) $x > y \Rightarrow xz > yz$
(d) $x > y$ and $y > z \Rightarrow x > z$

2. If $|4x + 3| > 7$ for $x \in R$, then the solution set is given by:
(a) $\{x \in R: 1 < x < -5/2\}$
(b) $\{x \in R: -1 < x < 5/2\}$
(c) $\{x \in R: x > 1\} \cup \{x \in R: x < -5/2\}$
(d) $\{x \in R: x < 1\} \cap \{x \in R : x > -5/2\}$

3. If x is a real number then solution set of $1 < (3x - 4)/8 < 4$ is:
(a) $4 < x < 12$
(b) $0 < x < 12$
(c) $4 < x < \infty$
(d) $-\infty < x < \infty$

4. The range of positive values of x which satisfy $5x + 2 < 3x + 8$ and $\frac{(x+2)}{(x-1)} < 4$ are:
(a) (2 , 3)
(b) $(0, \infty)$
(c) $(-\infty, 1)$
(d) (1, 3)

5. The solution set of an inequality $5 - 15y > 125$; $y \in R$ is :
(a) $\{y : y \in R\}$
(b) $\{y: y > 8\}$
(c) $\{y : y < 8\}$
(d) $\{y : y < -8\}$

6. If $x \in R$, then $|x| > x$ iff
(a) $x \ge 0$
(b) $x < 0$
(c) $x = 0$
(d) none of these

7. If $x, y \in R$ and $|x| + |y| = 0$, then
(a) $x > 0, y < 0$
(b) $x < 0, y > 0$
(c) $x = 0, y = 0$
(d) none of these.

8. Solution set of the inequation $\frac{1}{2x+1} > 1$ is

(a) $(-\infty, 0)$
(b) $\left(-\frac{1}{2}, 0\right)$
(c) $\left[-\frac{1}{2}, 0\right]$
(d) none of these.

9. Solution set of the inequations $\frac{x+1}{2} < \frac{2x+1}{3} < \frac{3x+2}{4}$ is
(a) R
(b) $(-2, \infty)$
(c) $(1, \infty)$
(d) $(-2, 1)$.

10. The solution of $| x - 3 | < 1$ is
(a) $-4 < x < -2$
(b) $-4 < x < 2$
(c) $-2 < x < 4$
(d) $2 < x < 4$.

11. Truth set of the inequation $| 3x | < | 6 - 3x|$ is
(a) $[1, \infty)$
(b) $(-\infty, 1)$
(c) $(-\infty, 1]$
(d) none the these

12. If $x, y \in R$ and $x > y \Rightarrow | x | > | y |$, then
(a) $x > 0$
(b) $y > 0$
(c) $x < 0$
(d) $y < 0$

13. If $2 < x < 3$, then
(a) $|x - 3| < |x - 2|$
(b) $(x - 3) > (x - 2)$
(c) $(x - 3)(x - 2) < 0$
(d) $\frac{x-3}{x-2} > 0$.

14. If $a, b, \in R$ and $a < b$, then
(a) $\frac{1}{a} < \frac{1}{b}$
(b) $\frac{1}{a} > \frac{1}{b}$
(c) $a^2 > b^2$
(d) nothing can be said

15. $| x - 1 | < | x + 1|$, where $x \in R$ is true only if
(a) $x > 0$
(b) $x > 1$
(c) $x < -1$ or $x > 0$
(d) x takes any value.

16. If $x \in R$, then $| x | =$
(a) x
(b) $-x$
(c) $\max \{x, -x\}$
(d) $\min \{x, -x\}$.

17. If $| 2x + 5 | < x + 3$, then x lies in
(a) $\left[-\frac{8}{3}, -2\right]$
(b) $\left(-\frac{8}{3}, -2\right)$
(c) $\left[-\frac{5}{2}, -2\right]$
(d) $\left[\frac{5}{2}, \frac{8}{3}\right]$.

18. The inequality $\frac{2}{x} < 3$ is true, when x belongs to
(a) $\left[\frac{2}{3}, \infty\right)$
(b) $\left(-\infty, \frac{2}{3}\right]$
(c) $\left(\frac{2}{3}, \infty\right) \cup (-\infty, 0)$
(d) none of these

19. $\frac{x+4}{x-3} < 2$ is satisfied when x satisfies

(a) $(-\infty, 3) \cup (10, \infty)$ (b) $(3, 10)$
(c) $(-\infty, 3) \cup [10, \infty)$ (d) none of these.

20. Solution of $\frac{x-7}{x+3} > 2$ is
(a) $(-3, \infty)$ (b) $(-\infty, -13)$
(c) $(-13, -3)$ (d) none of these.

21. Solution of $\frac{2x-3}{3x-5} \geq 3$ is
(a) $\left[1, \frac{12}{7}\right)$ (b) $\left(\frac{5}{3}, \frac{12}{7}\right]$
(c) $\left(-\infty, \frac{5}{3}\right)$ (d) $\left[\frac{12}{7}, \infty\right)$.

22. Solution of | 3x + 2 | < 1 is
(a) $\left[-1, -\frac{1}{3}\right]$ (b) $\left\{-\frac{1}{3,} - 1\right\}$
(c) $\left(-1, -\frac{1}{3}\right)$ (d) none of these.

23. Solution of |3x – 2| ≥ 1 is
(a) $\left[\frac{1}{3}, 1\right]$ (b) $\left(\frac{1}{3}, 1\right)$
(c) $\left\{\frac{1}{3}, 1\right\}$ (d) $\left(-\infty, \frac{1}{3}\right] \cup [1, \infty)$

24. Solution of | 3 – x | = x – 3 is
(a) x < 3 (b) x > 3
(c) x ≥ 3 (d) x ≤ 3.

25. Solution of $\left|\frac{1}{x} - 2\right| < 4$ is
(a) $\left(-\infty, -\frac{1}{2}\right)$ (b) $\left(\frac{1}{6}, \infty\right)$
(c) $\left(-\frac{1}{2}, \frac{1}{6}\right)$ (d) $\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{6}, \infty\right)$.

26. Solution of $\left|1 + \frac{3}{x}\right| > 2$ is
(a) (0, 3] (b) [–1, 0)
(c) $(-1, 0) \cup (0, 3)$ (d) none of these

27. Solution of | 2x – 3 | < | x + 2 | is
(a) $\left(-\infty, \frac{1}{3}\right)$ (b) $\left(\frac{1}{3}, 5\right)$
(c) $(5, \infty)$ (d) $\left(-\infty, \frac{1}{3}\right) \cup (5, \infty)$.

28. Solution of $0 < |3x + 1| < \frac{1}{3}$ is
(a) $\left(-\frac{4}{9}, -\frac{2}{9}\right)$ (b) $\left[-\frac{4}{9}, -\frac{2}{9}\right]$

(c) $\left(-\frac{4}{9},-\frac{2}{9}\right)-\left\{-\frac{1}{3}\right\}$ (d) $\left[-\frac{4}{9},-\frac{2}{9}\right]-\left\{-\frac{1}{3}\right\}$

29. Solution of $|x-1| \geq |x-3|$ is
(a) $x \leq 2$ (b) $x \geq 2$
(c) [1,3] (d) none of these.

30. The solution of $||x|-1| < |1-x|, x \in R$ is
(a) (–1, 1) (b) (0, ∞)
(c) (–1, ∞) (d) none of these.

Answers

Practice Assignment I

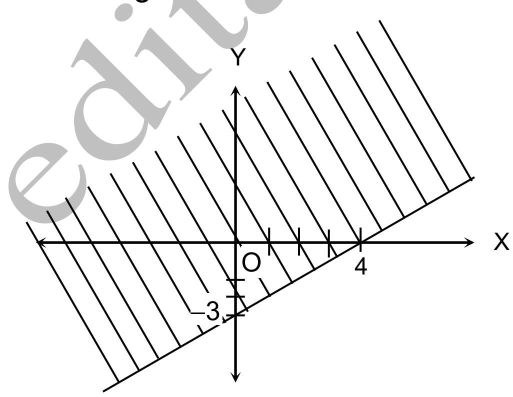
1. (i) { 1, 2, 3, 4} (ii) {……..–2, –1, 0, 1, 2, 3, 4} (iii) $(-\infty, 4\frac{1}{6})$
2. (i) { 1, 2, 3, 4} (ii) {……..–2, –1, 0, 1, 2, 3, 4} (iii) $(-\infty, 4]$
3. $\left(\frac{11}{5}, \infty\right)$
4. $[-7, \infty)$
5. $\left(\frac{-53}{14}, \infty\right)$
6. $[1, \infty)$
7. $(-\infty, 120]$
8. $(-\infty, 2)$
9. $(-\infty, 0) \cup (4, \infty)$
10. $\left(1, \frac{5}{3}\right)$
11. $(-\infty, -18] \cup (-7, \infty)$
12. $(-\infty, \frac{9}{14}) \cup (1, \infty)$
13. $[-4, 4]$
14. $(-\infty, -7] \cup [7, \infty)$
15. ϕ
16. $[-5, 6]$
17. $(2, \infty)$
18. $\left(\frac{3}{4}, \frac{7}{2}\right)$
19. $(-\infty, 0) \cup (1, \infty)$
20. $(-\infty, 0] \cup [4, \infty)$
21. $(-5, -2) \cup (-1, \infty)$
22. greater than or equal to 35 marks
23. 37 marks
24. 18 cm
25. (i) less than 10 hrs (ii) more than 10 hrs

Practice Assignment II

1. $[2, 6)$
2. ϕ
3. $(-\infty, -8.5)$
4. $(-\infty, -21)$
5. $(-\infty, 2)$
6. $(5, \infty)$
7. $(-\infty, -5.5)$
8. ϕ
9. $(1, 9)$
10. ϕ
11. (11, 13), (13, 15), (15, 17), (17, 19)
12. (10, 12), (12, 14)
13. Between 20°C and 25°C
14. Between 120 ℓ and 300 ℓ
15. Between 562.5 ℓ and 900 ℓ
16. Between 320 ℓ and 1280 ℓ

Practice Assignment III

1.



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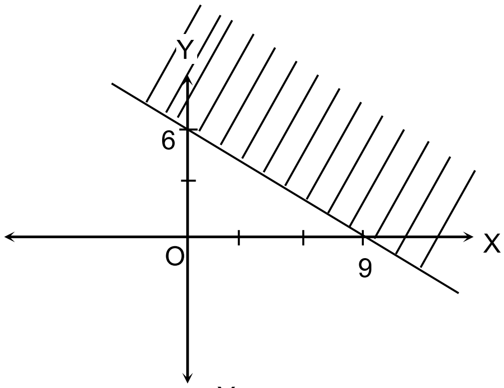

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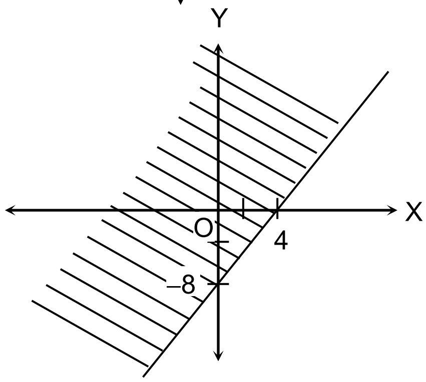

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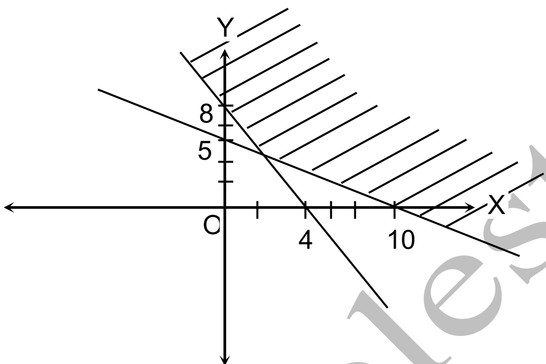

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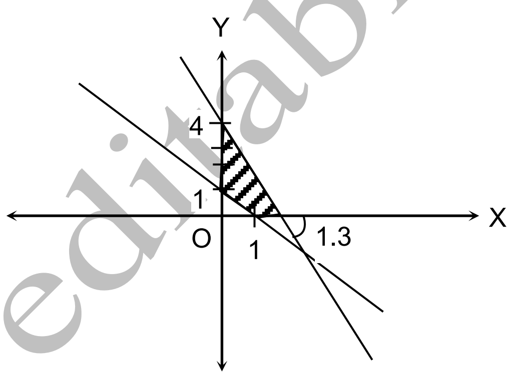
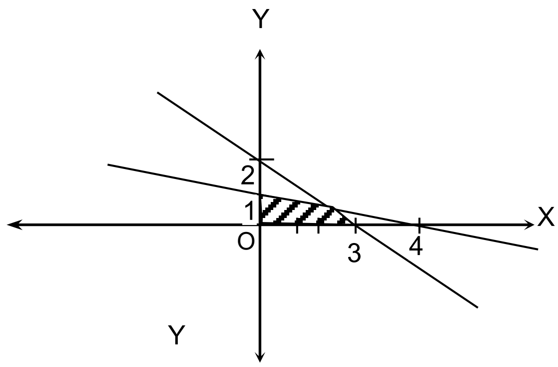


6. No solution

7.



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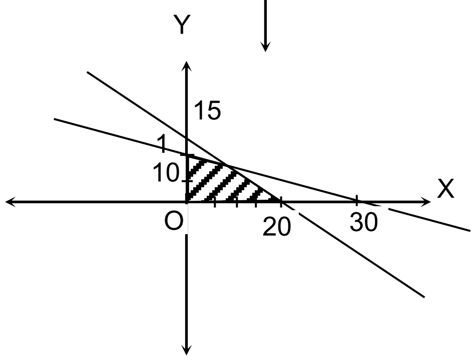

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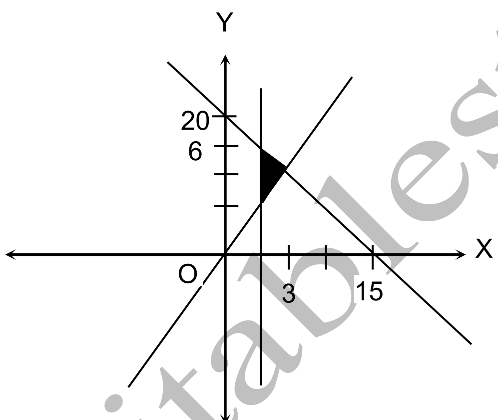

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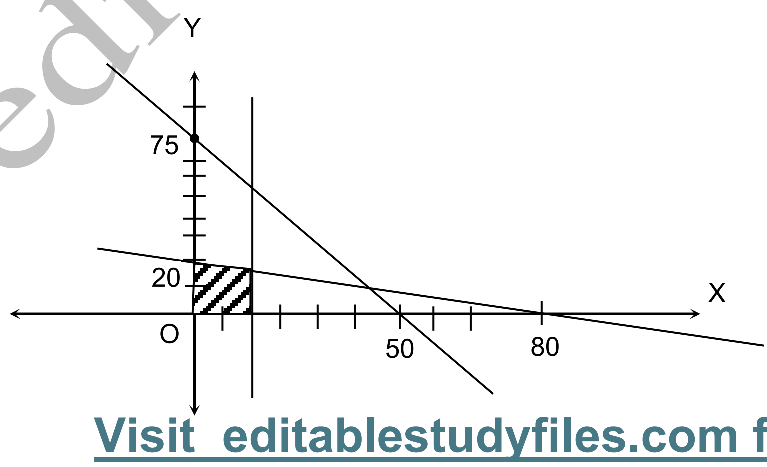

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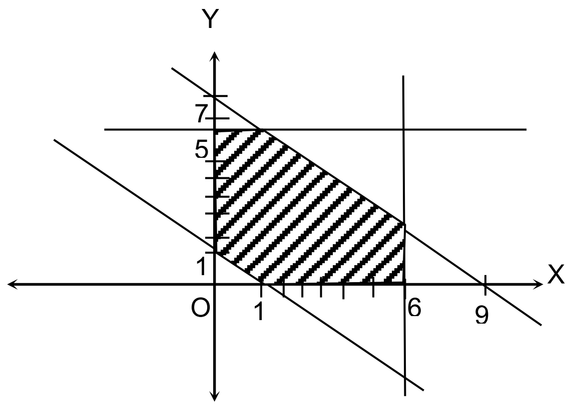

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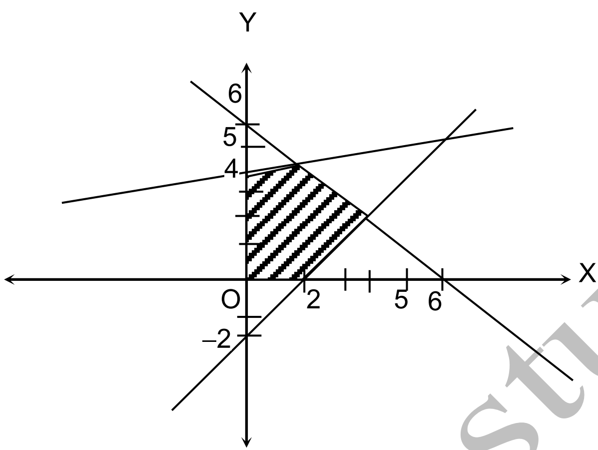

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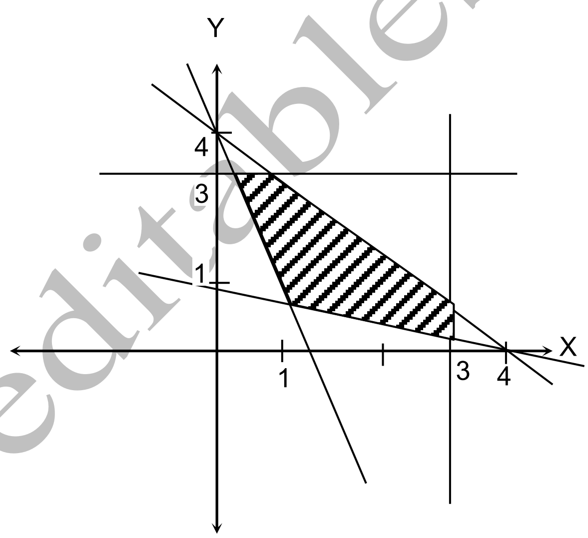

14. No solution

Objective Assignment

1	C	8	B	15	A	22	C	29	B
2	C	9	C	16	C	23	D	30	D
3	A	10	D	17	B	24	C		
4	A	11	B	18	C	25	D		
5	D	12	B	19	A	26	C		
6	B	13	C	20	C	27	B		
7	C	14	D	21	B	28	C		

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